Name: Key Hour: ________ Seat: ________

Section 8.1: Geometric Mean

Warm Up: What do you want to do differently this chapter? What do you want to keep doing?

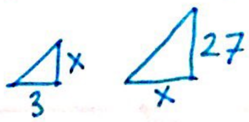
Objectives

1. Find the Geometric Mean between 2 #'s
2. Solve problems involving relationships between parts of a right triangle + the altitude to its hypotenuse

New Vocabulary

Geometric Mean(p. 537): the positive square root between the product of two numbers

? How do we solve: $\frac{3}{x} = \frac{x}{27}$



KeyConcept Geometric Mean

Words The geometric mean of two positive numbers a and b is the number x such that $\frac{a}{x} = \frac{x}{b}$. So, $x^2 = ab$ and $x = \sqrt{ab}$.

Example The geometric mean of $a = 9$ and $b = 4$ is 6, because $6 = \sqrt{9 \cdot 4}$.

Arithmetic Mean	Geometric mean
· Add all numbers	· Multiply all numbers
· Divide by the number of terms	· Take the n^{th} root of the n numbers.
ex: 3, 4, 5	ex: 3, 4 5

Average vs. Geometric Mean

Average	Geometric Mean
The average of two numbers, x and y: $\frac{x+y}{2}$	The geometric mean (x) of two number, a and b: $\sqrt{a \cdot b}$

EXAMPLE 1 Geometric Mean

Find the geometric mean between 8 and 10.

Work:

$$\text{Geo Mean} = \sqrt{8 \cdot 10} = \sqrt{80} = \sqrt{16 \cdot 5} = 4\sqrt{5}$$

$$\approx 8.9$$

EXAMPLE 1 Check Your Progress

Find the geometric mean between 5 and 45.

Work:

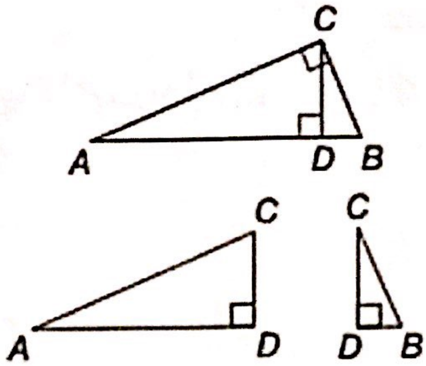
$$\sqrt{5 \cdot 45} = \sqrt{225} = 15$$

Answer: D

Theorem 8.1

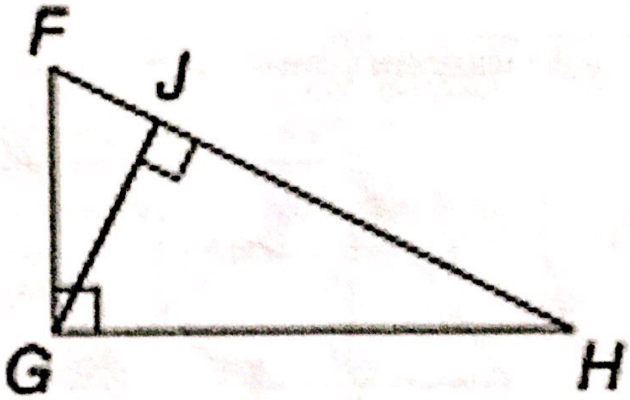
If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other.

Example If $\overline{CD}$ is the altitude to hypotenuse $\overline{AB}$ of right $\triangle ABC$, then $\triangle ACD \sim \triangle ABC$, $\triangle CBD \sim \triangle ABC$, and $\triangle ACD \sim \triangle CBD$.



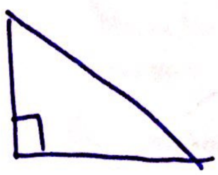
EXAMPLE 2 Identify Similar Right Triangles

Write a similarity statement identifying the three similar right triangles in the figure.

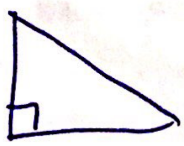


<u>**Reorienting Triangles**</u>: Draw the 3 triangles below that come from the above figure. Then write the similarity statement.

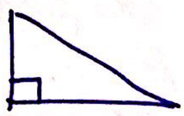
FGH



FJG



GJH

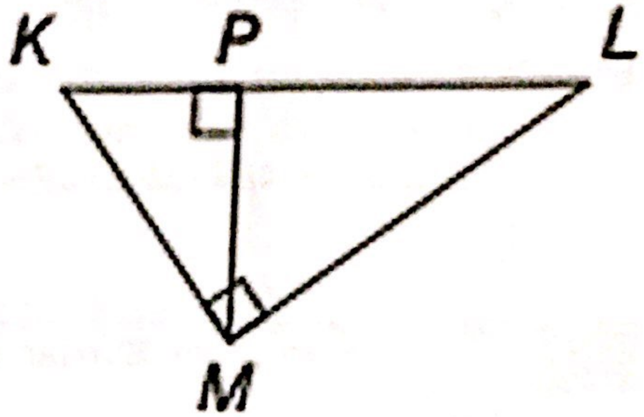


$\triangle FGH \sim \triangle FJG \sim \triangle GJH$

EXAMPLE 2 Check Your Progress

Work:

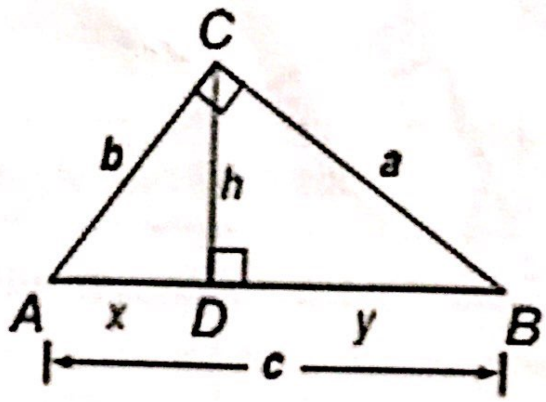
$\triangle KML \sim \triangle KPM \sim \triangle MPL$



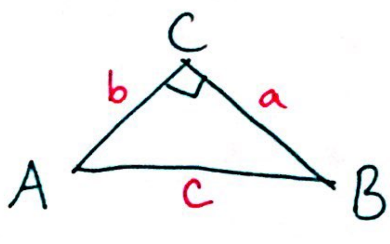
Answer: A

Let's Look Deeper...

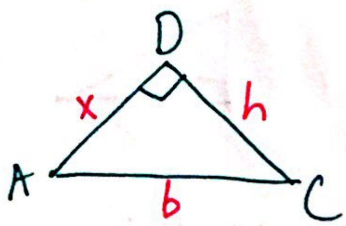
In the space below, draw triangles ACB, ADC, and CDB, labeling their sides and angles that you know.



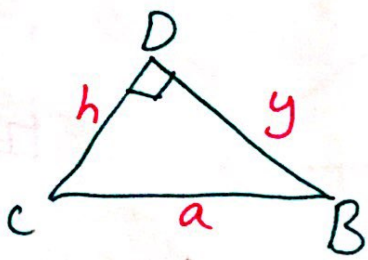
ACB



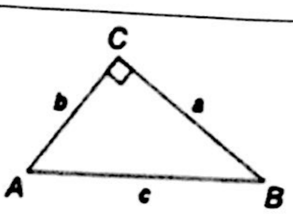
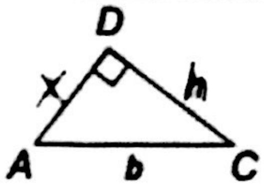
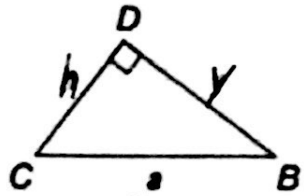
ADC



CDB

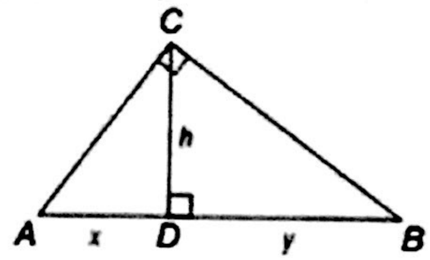


What do these proportions look like?

In each triangle, find the corresponding sides asked for in each ratio			
$\frac{\text{shorter leg}}{\text{longer leg}}$	$\frac{b}{a}$	$\frac{x}{h}$	$\frac{h}{y}$
$\frac{\text{hypotenuse}}{\text{shorter leg}}$	$\frac{c}{b}$	$\frac{b}{x}$	$\frac{a}{h}$
$\frac{\text{hypotenuse}}{\text{longer leg}}$	$\frac{c}{a}$	$\frac{b}{h}$	$\frac{a}{y}$

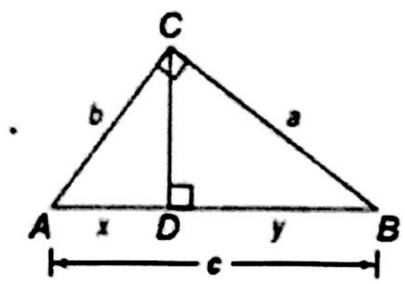
Theorems **Right Triangle Geometric Mean Theorems**

8.2 Geometric Mean (Altitude) Theorem The altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments. The length of this altitude is the geometric mean between the lengths of these two segments.



Example If $\overline{CD}$ is the altitude to hypotenuse $\overline{AB}$ of right $\triangle ABC$, then $\frac{x}{h} = \frac{h}{y}$ or $h = \sqrt{xy}$.

8.3 Geometric Mean (Leg) Theorem The altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments. The length of a leg of this triangle is the geometric mean between the length of the hypotenuse and the segment of the hypotenuse adjacent to that leg.



Example If $\overline{CD}$ is the altitude to hypotenuse $\overline{AB}$ of right $\triangle ABC$, then $\frac{c}{b} = \frac{b}{x}$ or $b = \sqrt{xc}$ and $\frac{c}{a} = \frac{a}{y}$ or $a = \sqrt{yc}$.

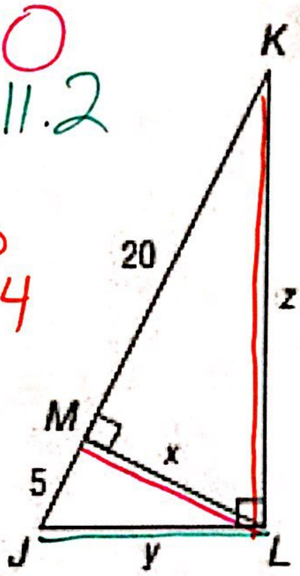
Example 3: Use Geometric Mean with Right Triangles

Find x, y, and z.

$x = \text{GeoMean}(20, 5) = \sqrt{20 \cdot 5} = \sqrt{100} = 10$

$y = \text{GeoMean}(5 \cdot (20+5)) = \sqrt{5 \cdot 25} = \sqrt{125} \approx 11.2$

$z = \text{GeoMean}(20, (25)) = \sqrt{20 \cdot 25} = \sqrt{500}$
≈ 22.4



Check Your Progress Example 3:

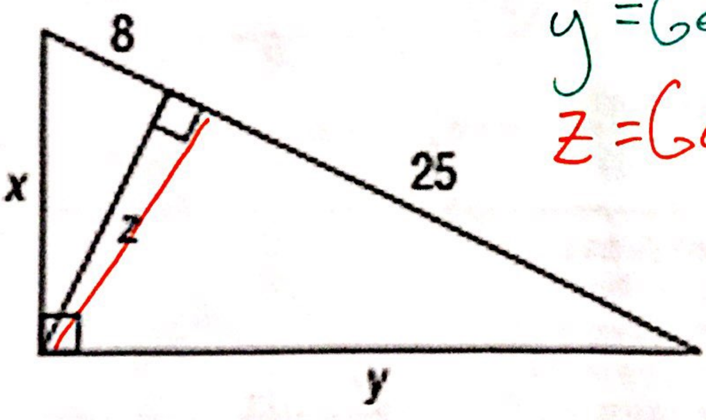
Use Geometric Mean with Right Triangles

Find x, y, and z.

$x = \text{GeoMean}(8, 33) = \sqrt{8 \cdot 33} \approx 16.25$

$y = \text{GeoMean}(25, 33) = \sqrt{25 \cdot 33} \approx 28.7$

$z = \text{GeoMean}(8, 25) = \sqrt{8 \cdot 25} \approx 14.1$



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